Mathematics and the Brain

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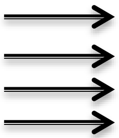
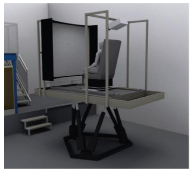
Dublin Institute of Technology

My background

- Numerical Analysis (Trinity College Dublin, PhD work)
  - Robust Numerical methods of Prandtl Boundary Layer Problems



- Self-motion Perception (Max Planck Institute for Biological Cybernetics)
  - Walking
  - Driving



- Unisensory and Multisensory processing
  - Developmental Disorders (Albert Einstein College of Medicine)
    - Autism Spectrum Disorder, Niemann Pick Type C
  - Movement Disorders (Trinity Centre for Bioengineering)
    - Parkinson's Disease
    - Dystonia

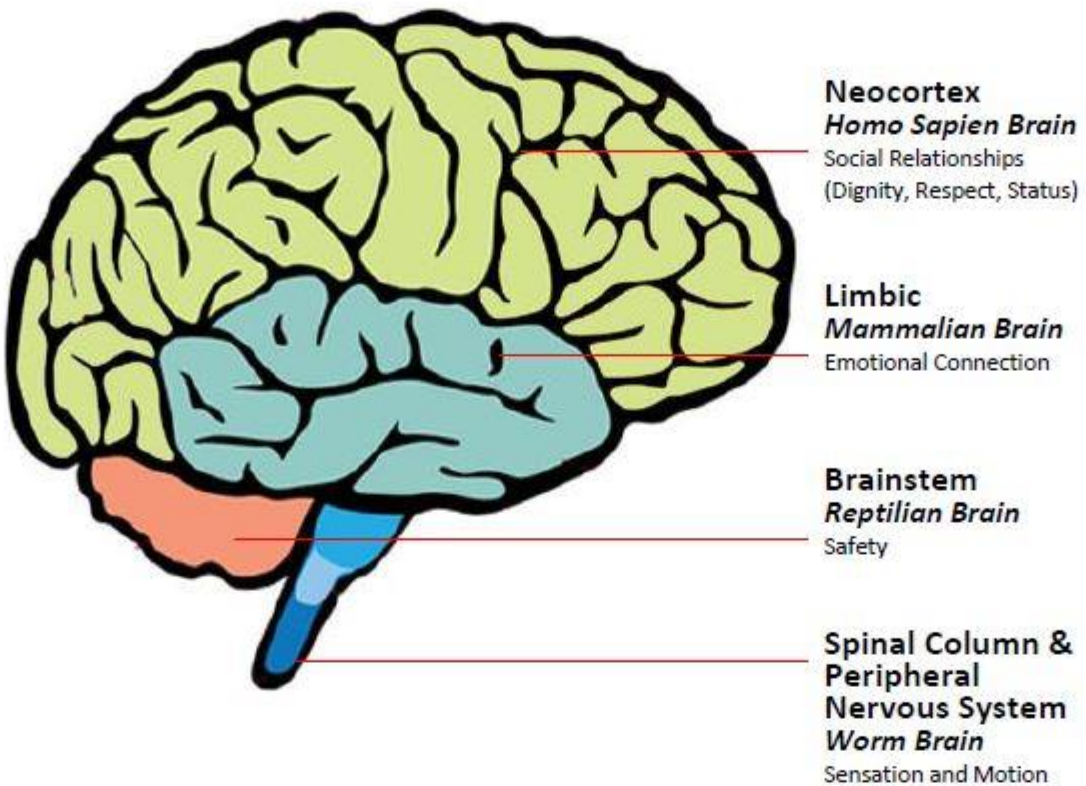
The Brain

What does the Brain do?

- Decision Making
- Emotions
- Memory
- Speech
- Dreams
- Movement
- Reactions
- Sensory processing
- Visual and Auditory Illusions
- Fight or flight

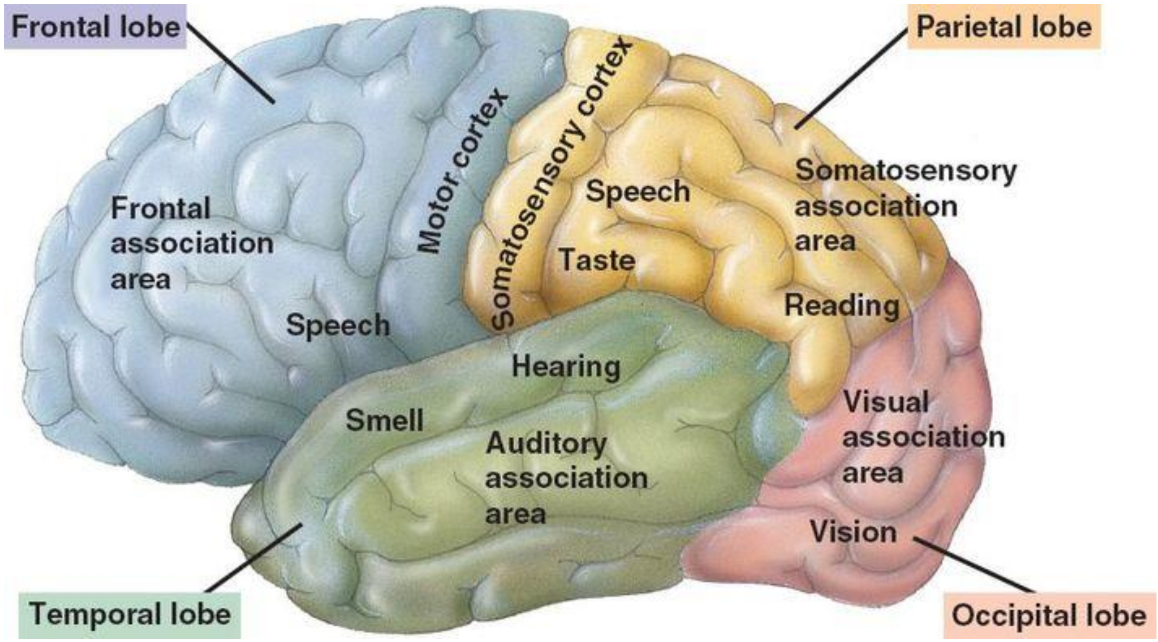
The Brain Song

The Brain



Neocortex
Homo Sapien Brain
Social Relationships
(Dignity, Respect, Status)
Limbic
Mammalian Brain
Emotional Connection
Brainstem
Reptilian Brain
Safety
Spinal Column & Peripheral Nervous System
Worm Brain
Sensation and Motion

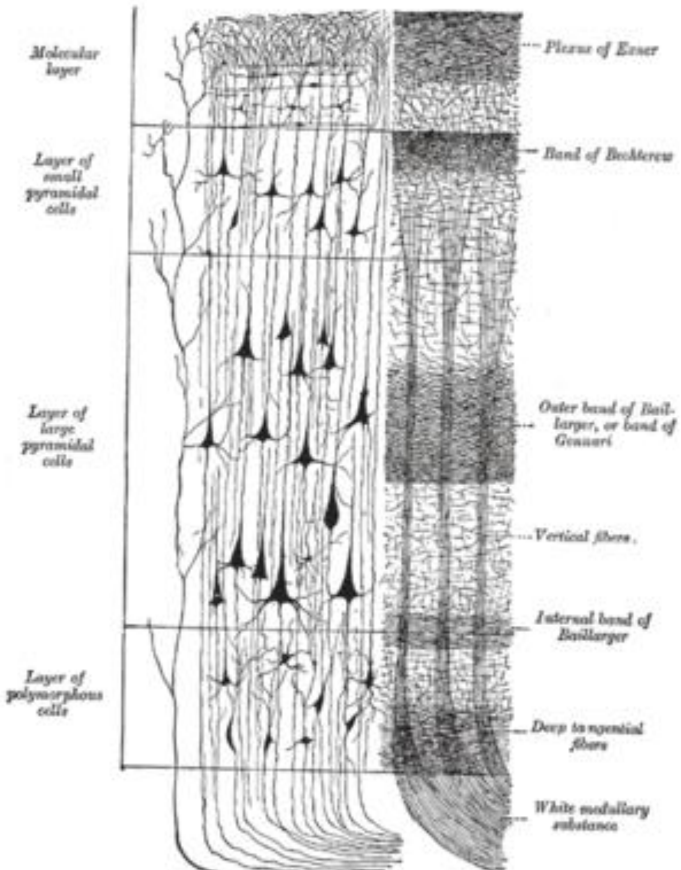
Neocortex



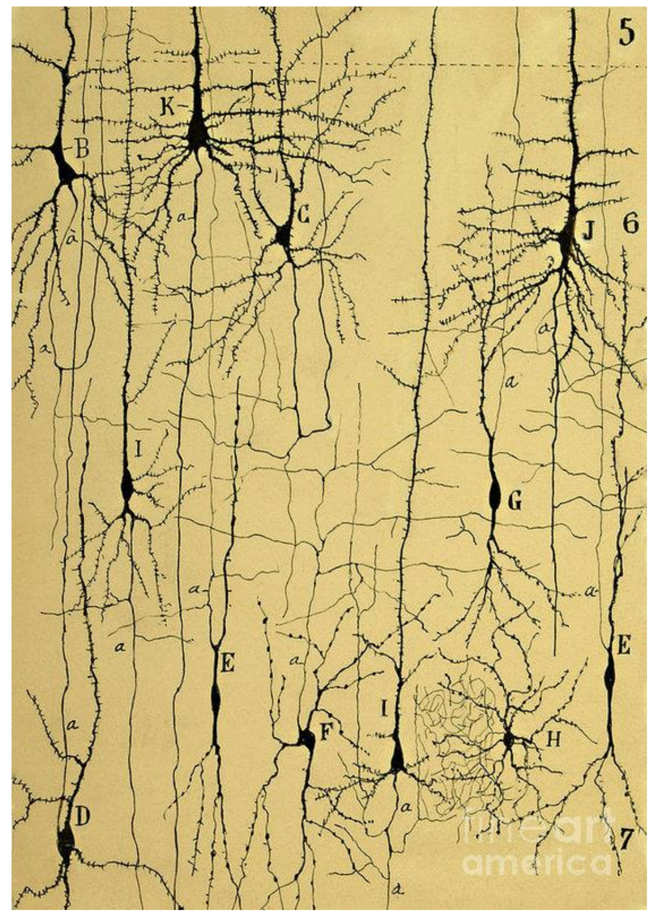
Frontal lobe
Parietal lobe
Motor cortex
Somatosensory cortex
Frontal association area
Speech
Taste
Somatosensory association area
Speech
Reading
Hearing
Smell
Auditory association area
Visual association area
Vision
Temporal lobe
Occipital lobe

Cortical Columns

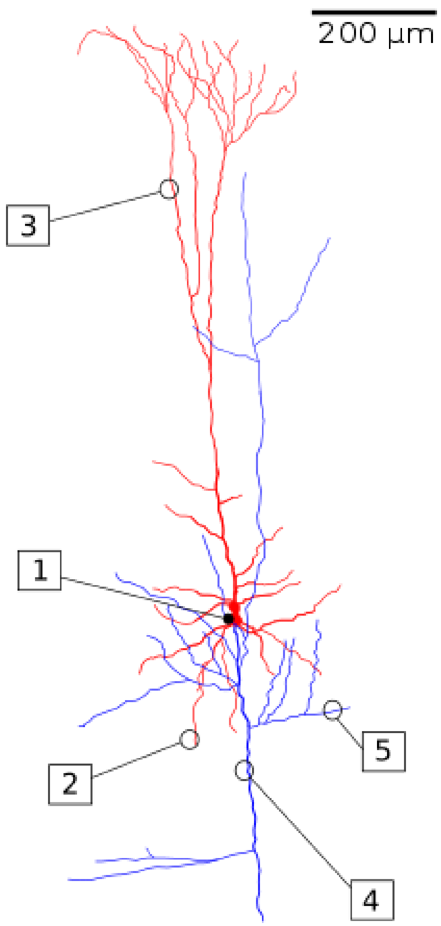
Cortical Layers

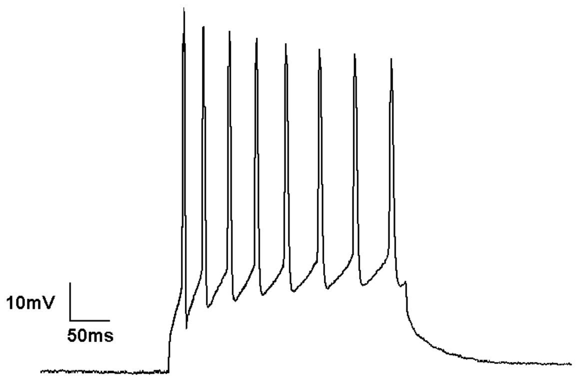


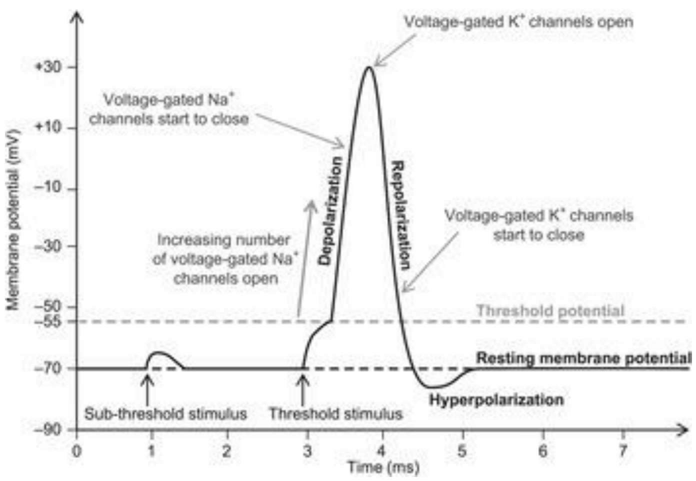
Columns of Neurons



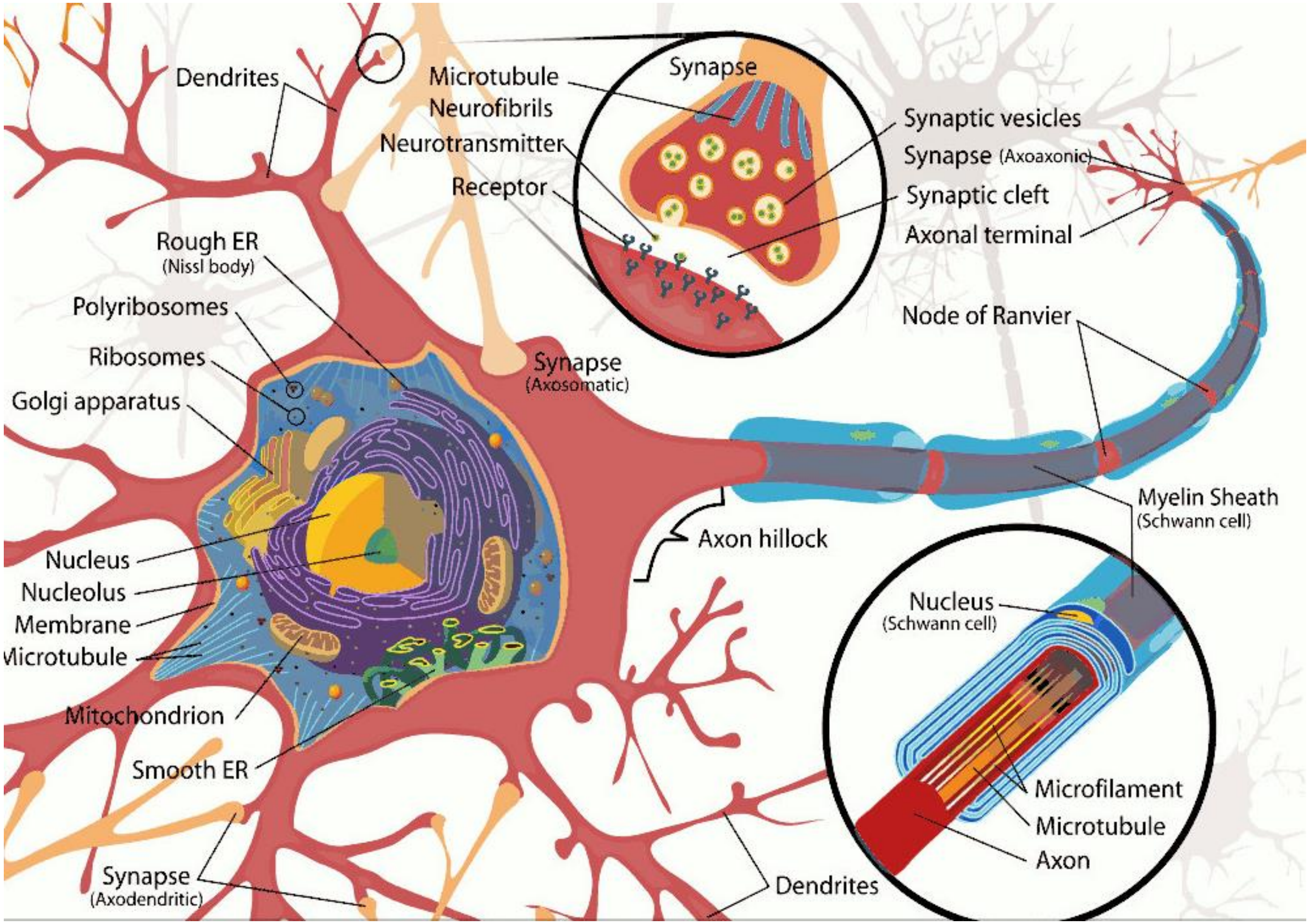
How do Neurons communicate







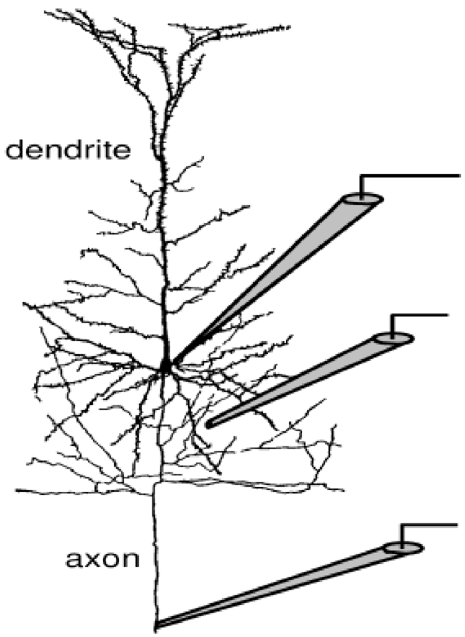
How does a Neuron generate action potentials



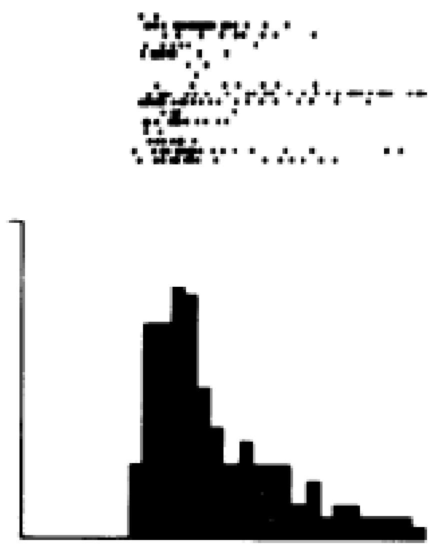
How do we investigate the brain

How do we record from Neurons?

Single Cell Recording



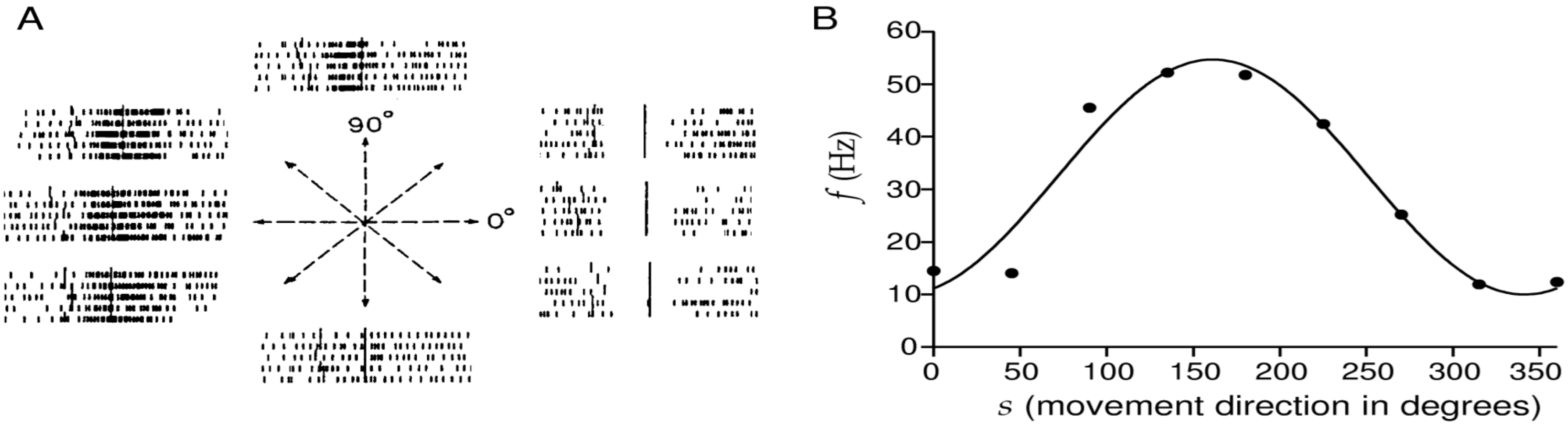
Multiple repetitions



How do we record from Neurons?

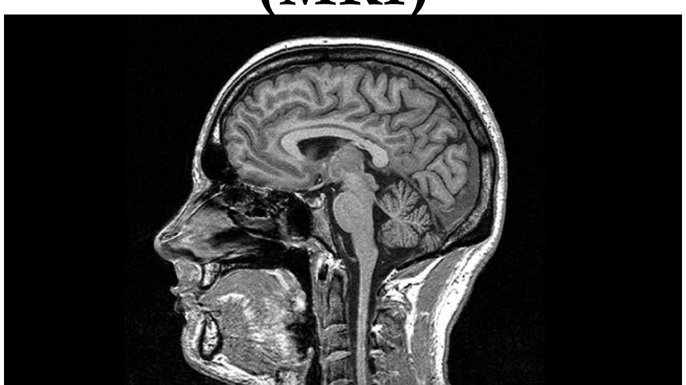
Hand reaching direction

Tuning curve of a motor cortical neuron

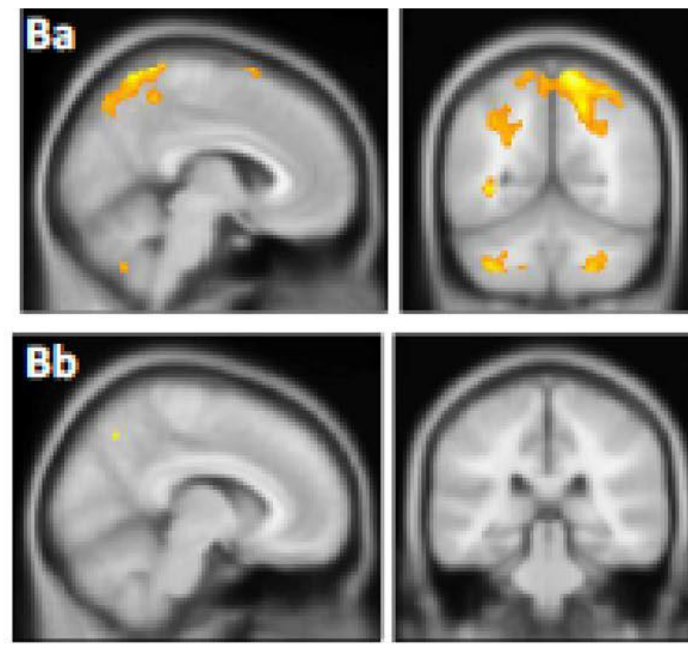


How do we record from cortical columns

Magnetic Resonance Imaging (MRI)

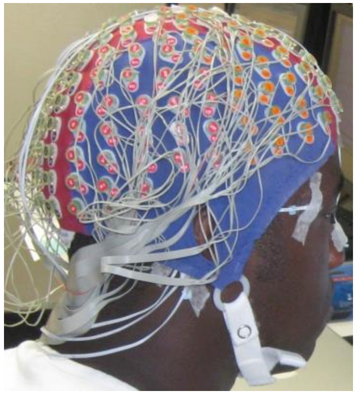


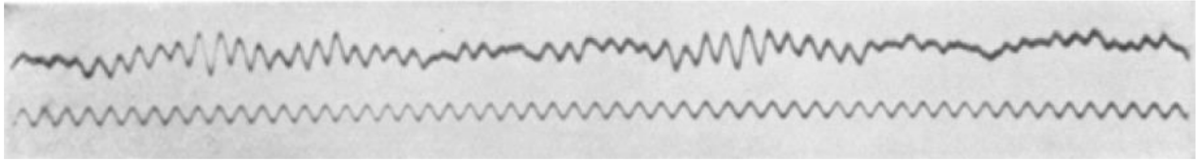
functional MRI(fMRI)



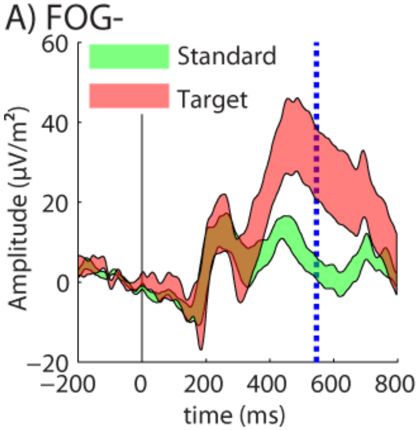
How do we record from cortical columns

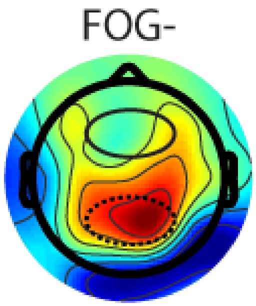
Electroencephalogram (EEG)





Event Related Potentials

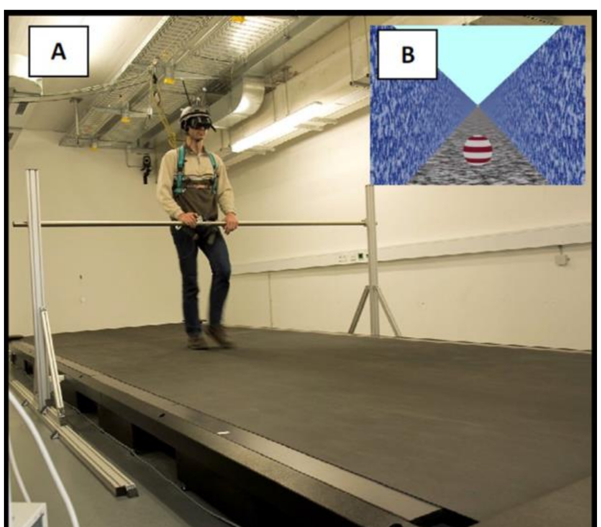


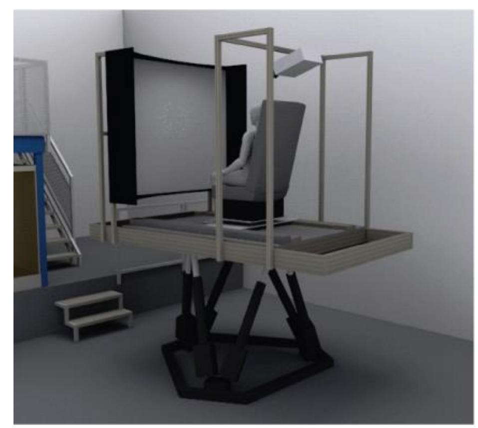


How do we study behaviour

- Questionnaires
- Reaction Time
- Response Time
- Choice
- Memory task
- Opinions
- Virtual Reality

Virtual Reality





How much Maths do we know?

What Maths do we know

- Add
- Subtraction
- Multiply
- Trigonometry
- Angles

} Primary School

- Probability
- Complex Numbers
- Differentiation

} Secondary School

- Integration
- Differential Equations
- Bayesian Statistics

} University

What Maths does our brain use

- Add
- Subtraction
- Multiply
- Trigonometry
- Angles

} Neurons

- Probability
- Complex Numbers
- Differentiation

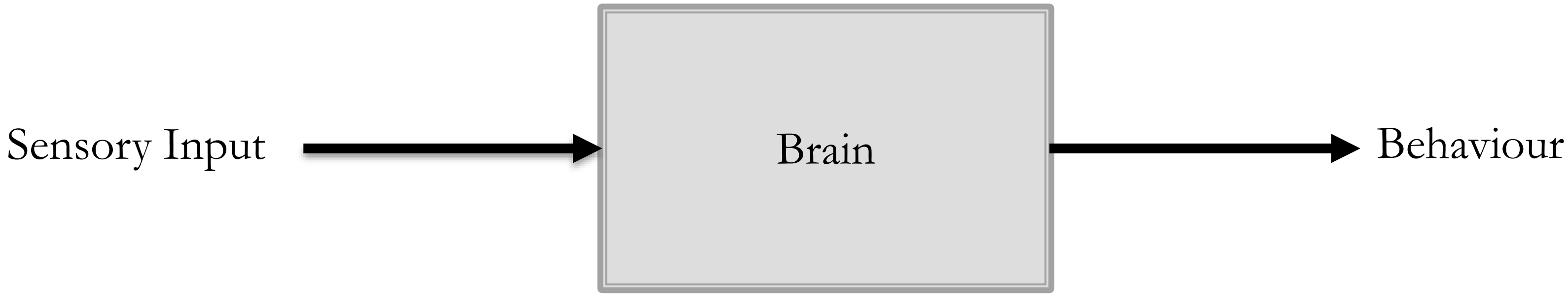
} Collection of neurons

- Integration
- Differential Equations
- Bayesian Statistics

} Cortical areas

How do we model the Brain?

How a Mathematician starts with the Brain



What Maths do we need to model the brain

- Add
- Subtraction
- Multiply
- Trigonometry
- Angles

- Probability
- Complex Numbers
- Differentiation

- Integration
- Differential Equations
- Bayesian Statistics

Mathematics and Neuroscience

- Analyse and Model
  - Chemical reactions (micro)
  - Neuronal Activity (micro)
  - Cortical Activity (mezzo)
  - Behaviour (macro)

- Goal to understand
  - Development
  - Combination of sensory signals
  - Movement
  - Learning
  - Diseases

Mathematics and Neuroscience

- Analyse and Model
  - Chemical reactions (micro)
  - Neuronal Activity (micro)
  - Cortical Activity (mezzo)
  - Behaviour (macro)

- Goal to understand
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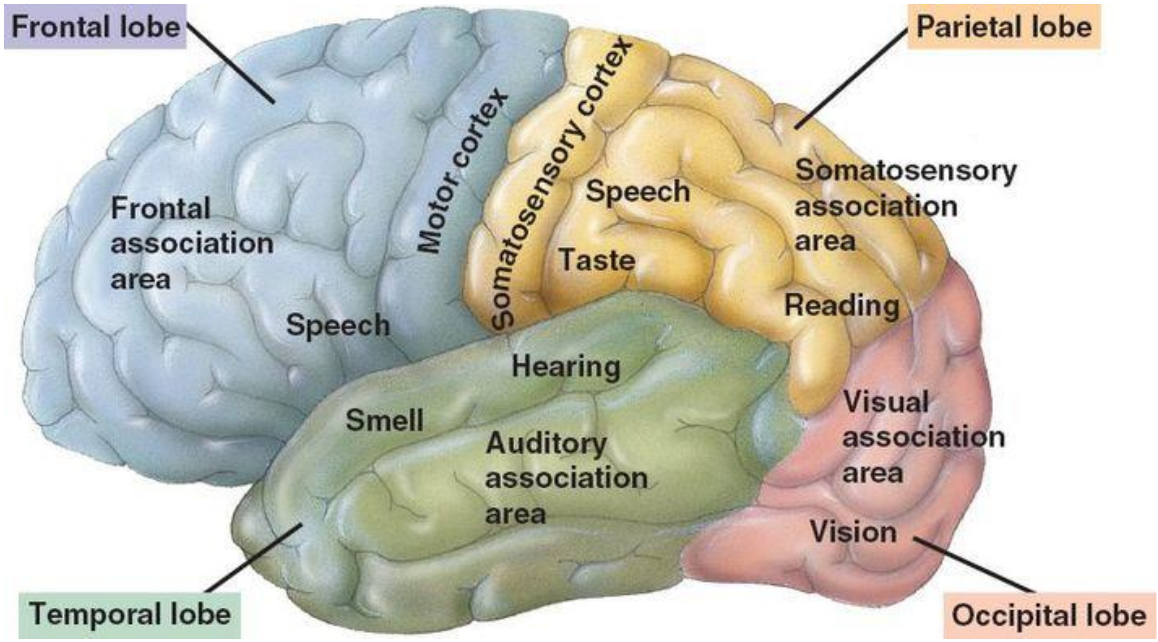
Multisensory Integration

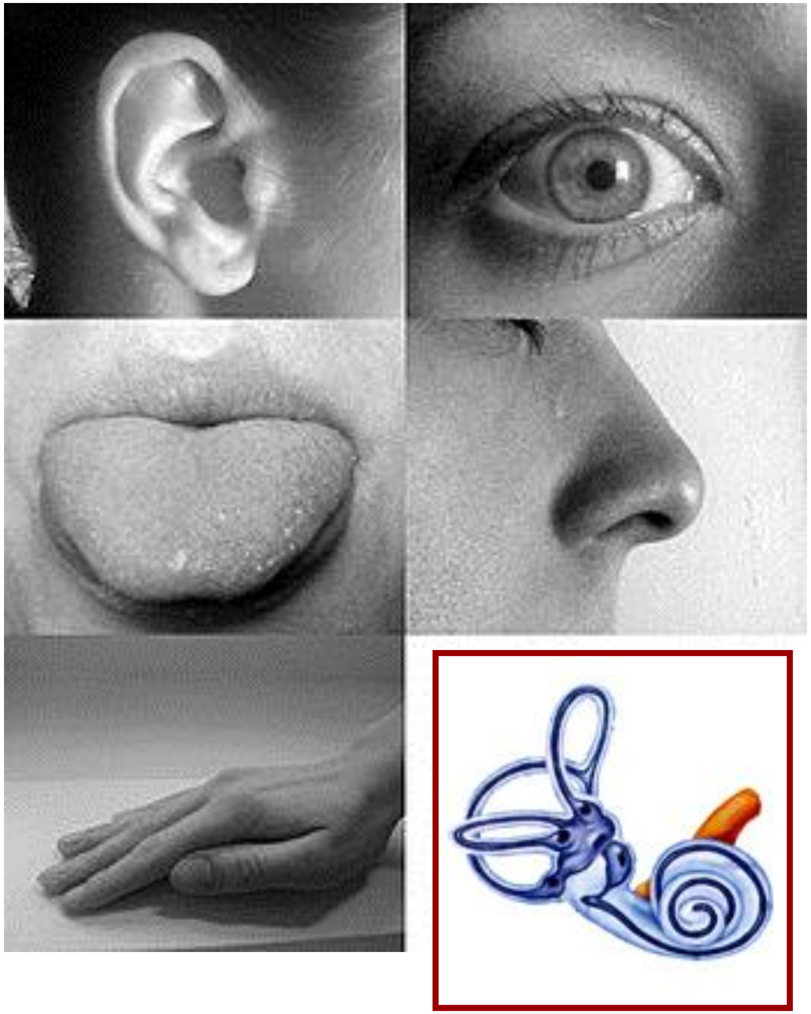
Neocortex



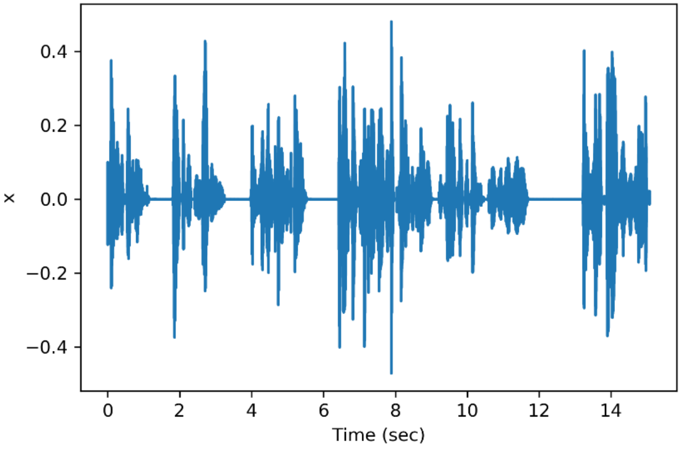
Frontal lobe
Parietal lobe
Motor cortex
Somatosensory cortex
Speech
Frontal association area
Taste
Somatosensory association area
Speech
Reading
Hearing
Smell
Auditory association area
Visual association area
Vision
Temporal lobe
Occipital lobe

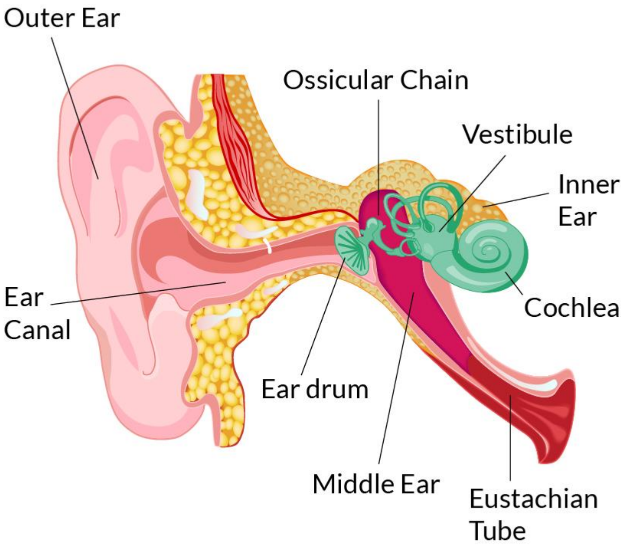
Sensory information

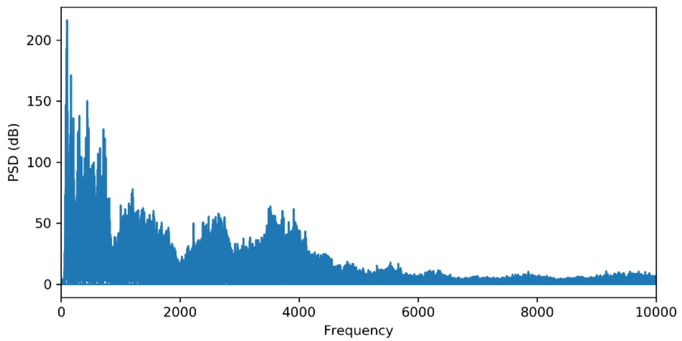
- Taste
- Smell
- Hearing
- Touch
- Sight
- Vestibular

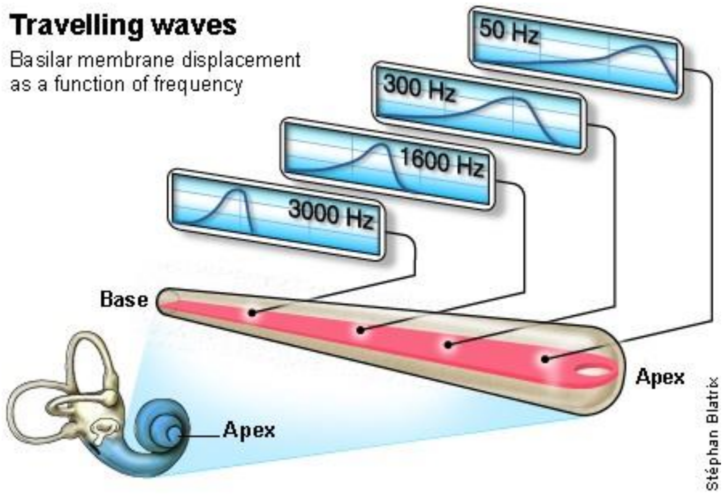


Hearing

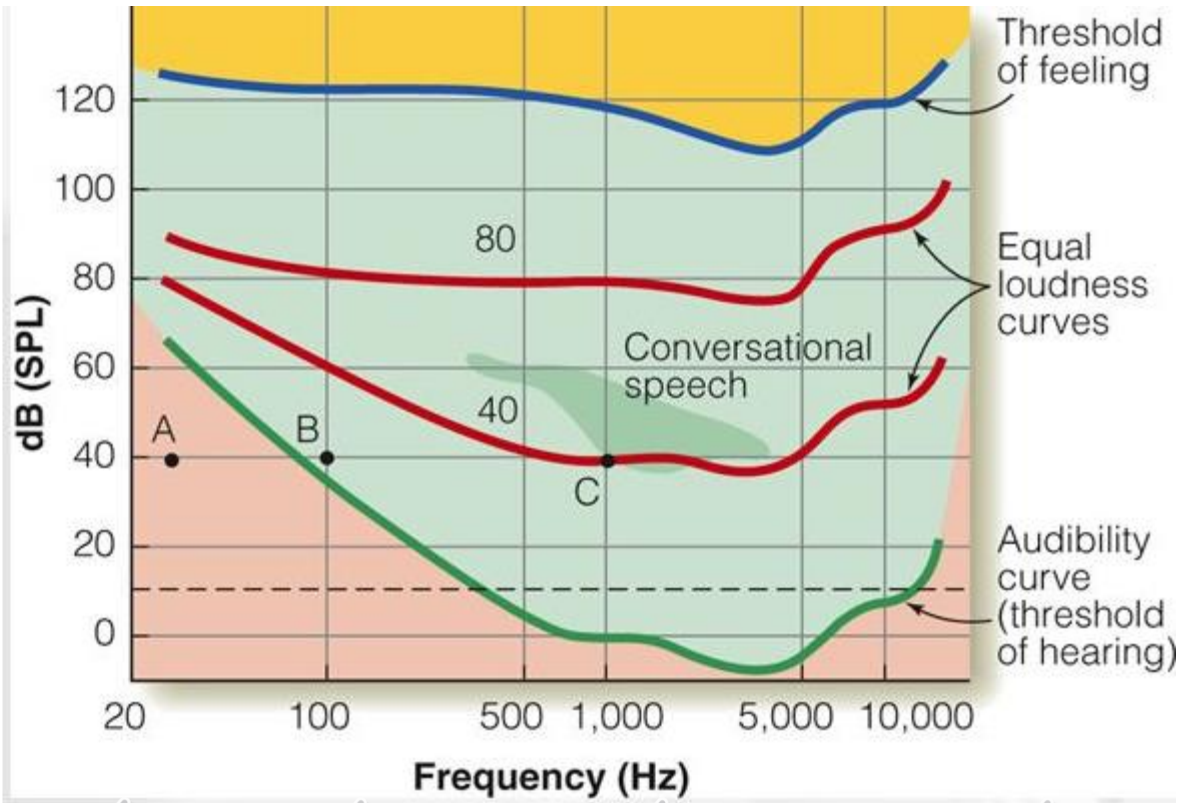

0.4
0.2
0.0
−0.2
−0.4
x
0
2
4
6
8
10
12
14
Time (sec)


Outer Ear
Ossicular Chain
Vestibule
Inner Ear
Ear Canal
Cochlea
Ear drum
Middle Ear
Eustachian Tube


200
150
100
50
0
PSD (dB)
0
2000
4000
6000
8000
10000
Frequency


Travelling waves
Basilar membrane displacement
as a function of frequency
50 Hz
300 Hz
1600 Hz
3000 Hz
Base
Apex
Apex
Stéphan Blatrix

Audio information






Multisensory Integration

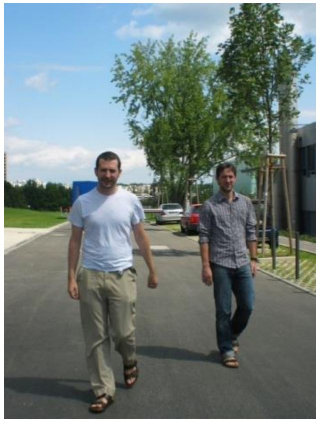
- Speech (Audio, Visual)
- Eating (Visual, Auditory, Smell)
- Rainbow (Visual, Touch)
- Cooking (Visual, Touch, Smell)
- Music (Auditory, Touch, Vestibular)
- Walking (Visual, Vestibular, Touch, Auditory)
- Everything is multisensory

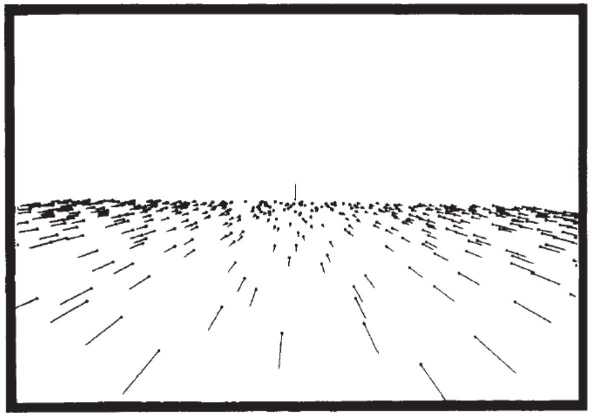
The Development Trajectory of Multisensory Integration

Childhood	Adulthood	Late Adulthood
Linear	Integration	Super Integration
Learns from more accurate modality Uses Fastest modality	Optimal Integration Faster Reaction times	Faster Reaction Times Susceptible to illusions

Self-motion

- Self-motion
  - Walking
  - Driving
- Cues for Self-motion
  - Visual
  - Vestibular
  - Touch
  - Audio
  - Etc.





Optic flow (visual)

Behavioural

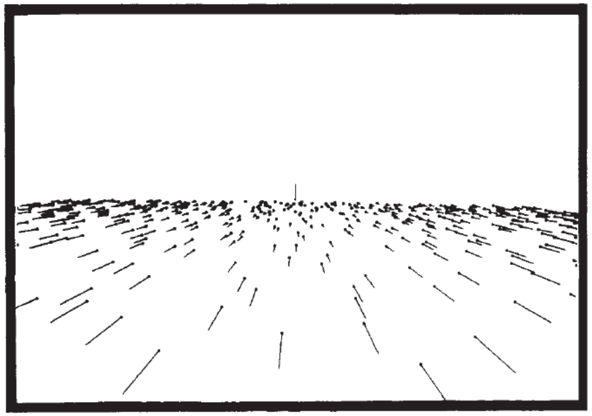
- Relative distance perception
- Heading
- Speed

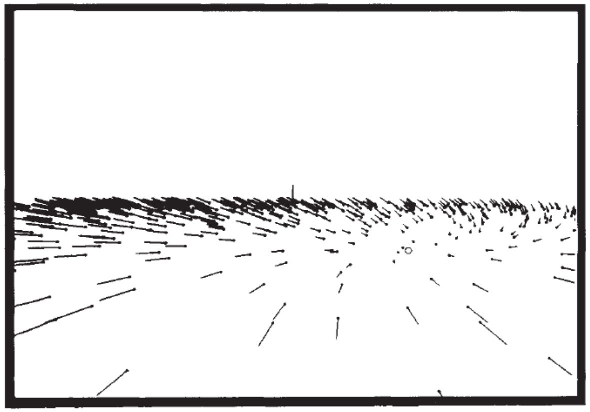
Function

- Balance
- Object motion
- Self-motion

Disorders

- Monopic vision



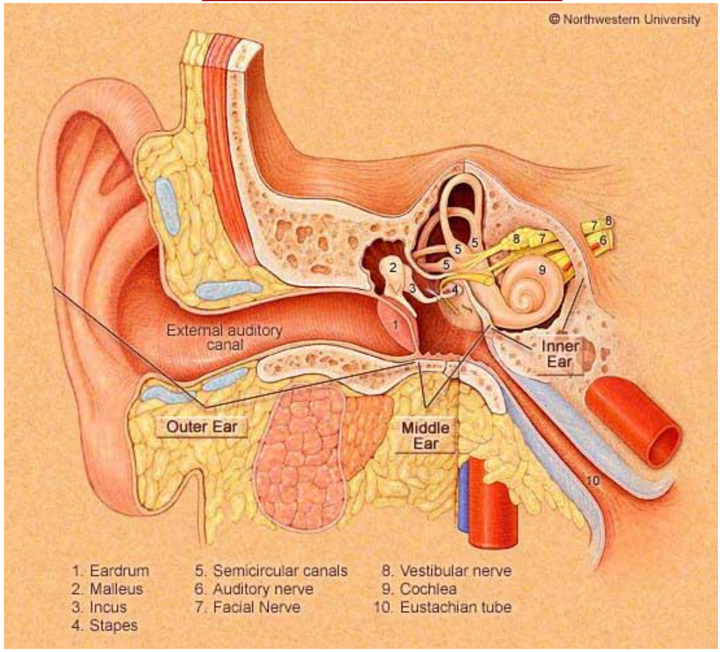


Inertial (vestibular)

- Otholiths
  - Linear acceleration
- Semi-circular Canals
  - Rotational velocity

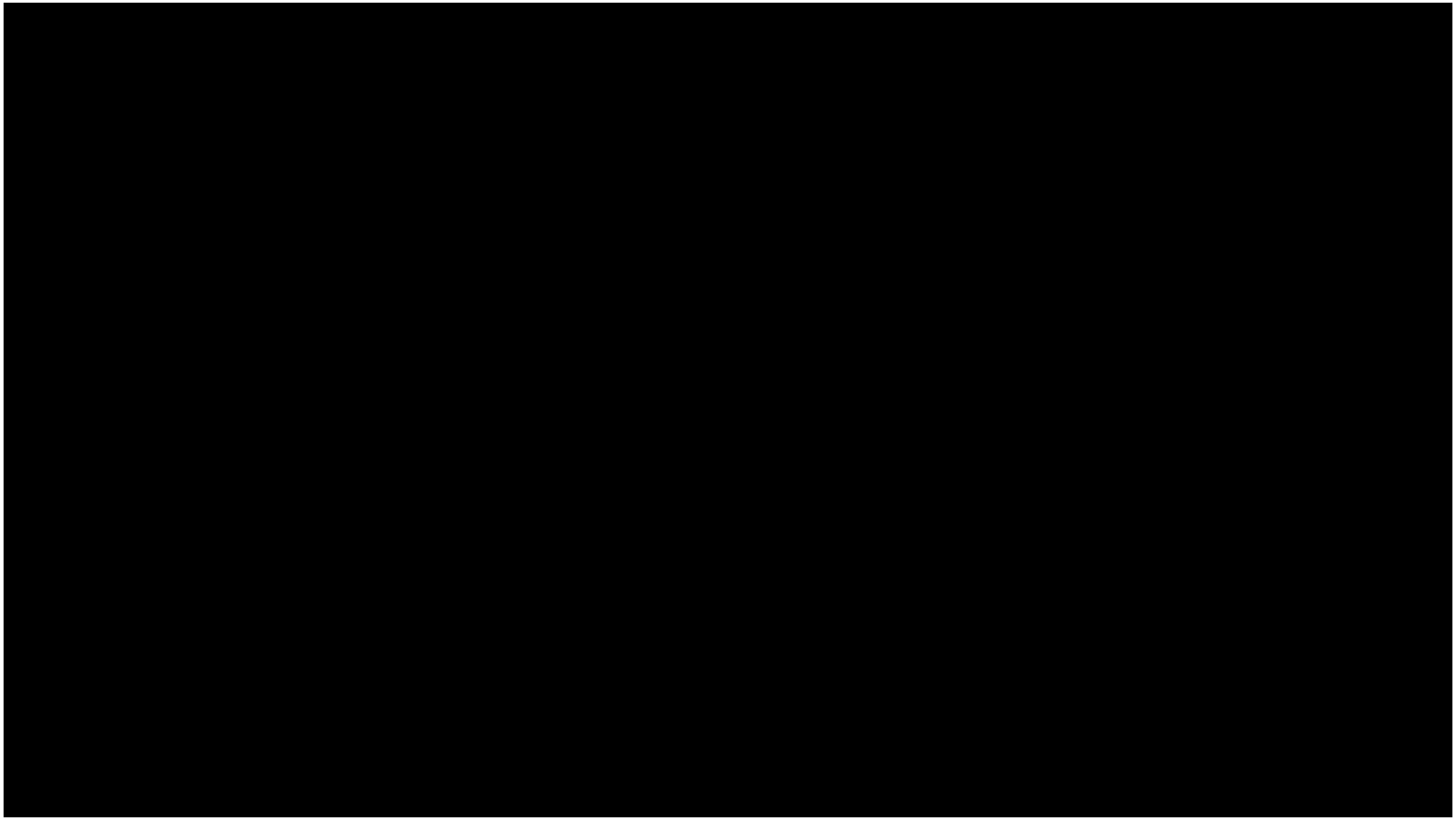
- Function
  - Eye movements
  - Heading
  - Gravity

- Disorders
  - Vertigo
  - Motion sickness
  - Falls



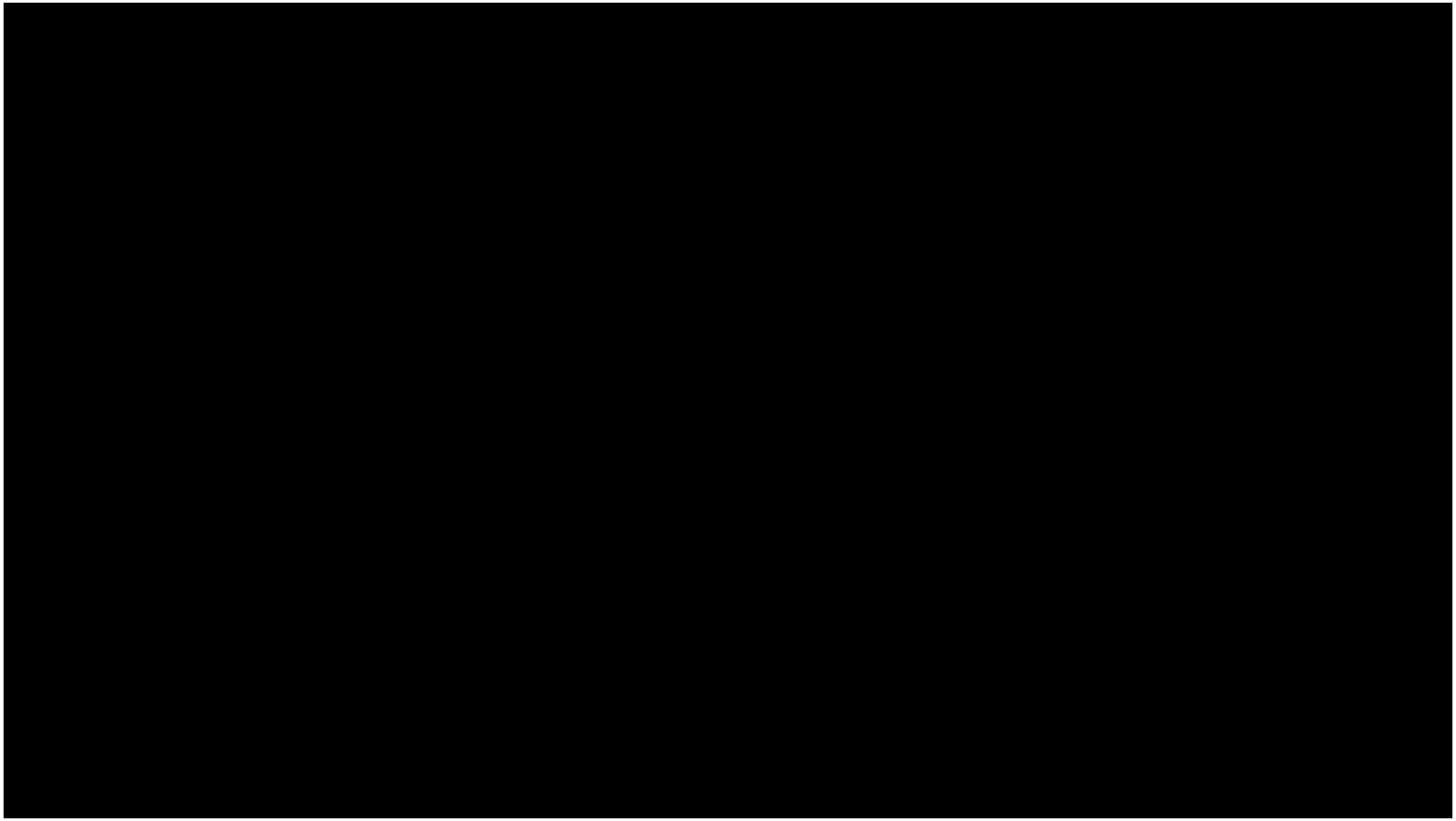
Vestibular

Eye and Head Movements



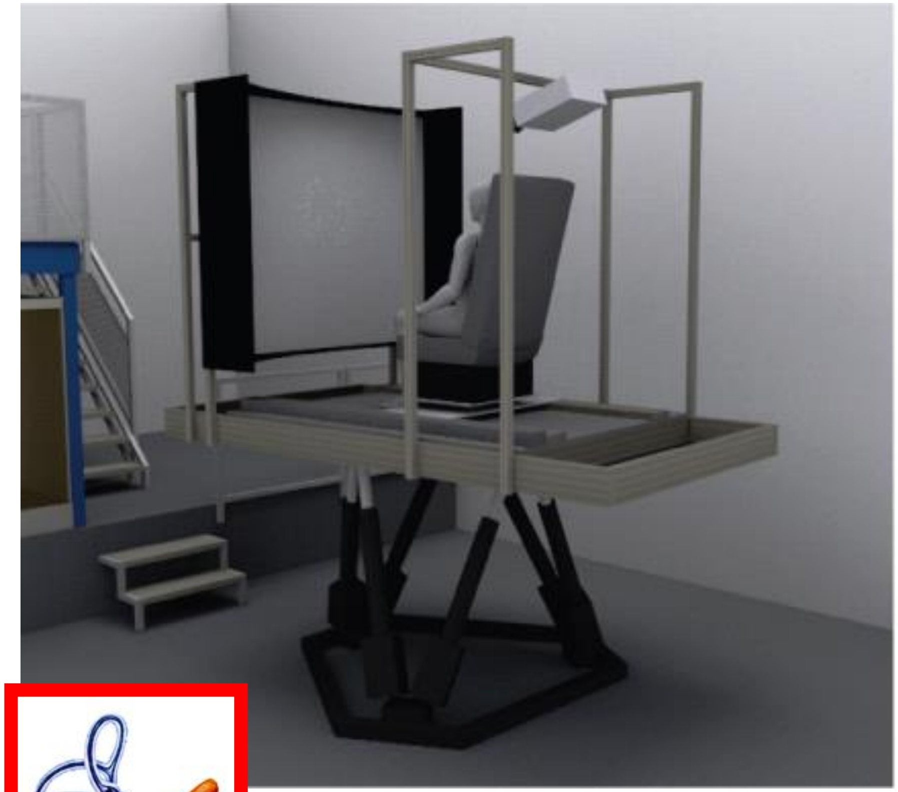
Vestibular illusions

Falling

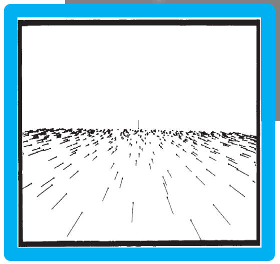


Virtual reality setup and stimuli

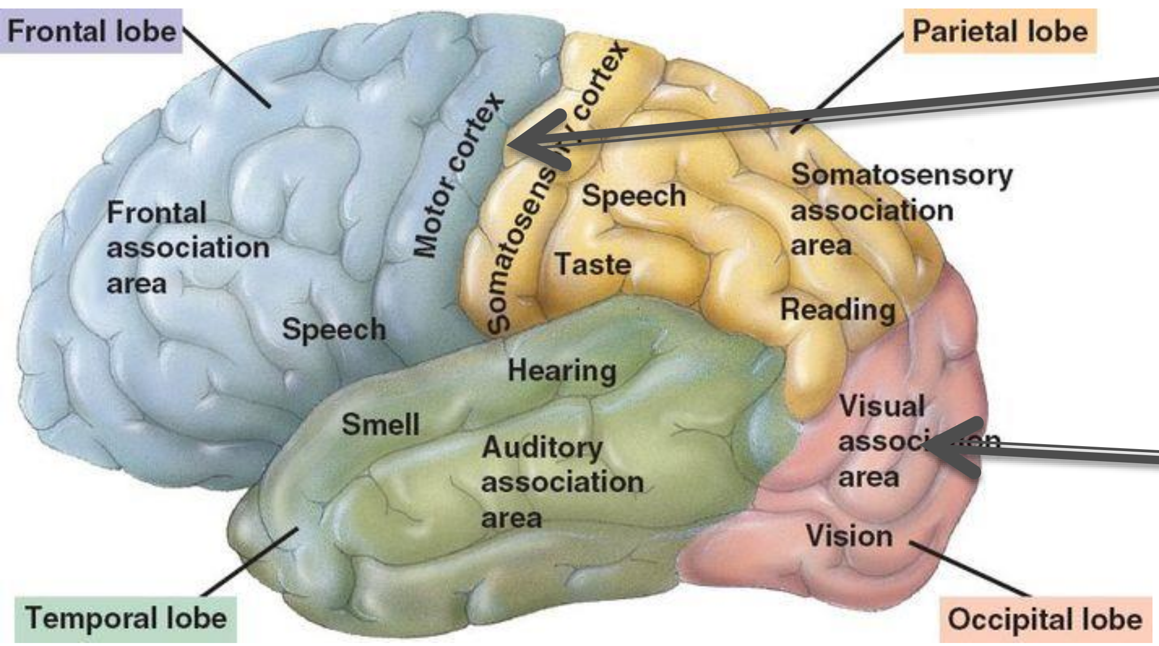
Motion Platform

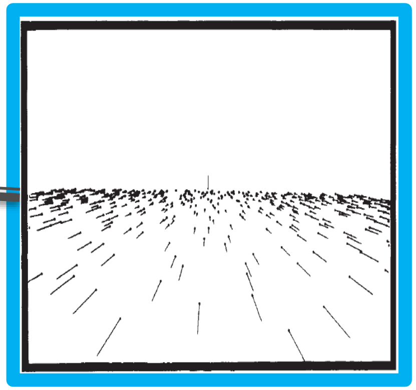


Visual



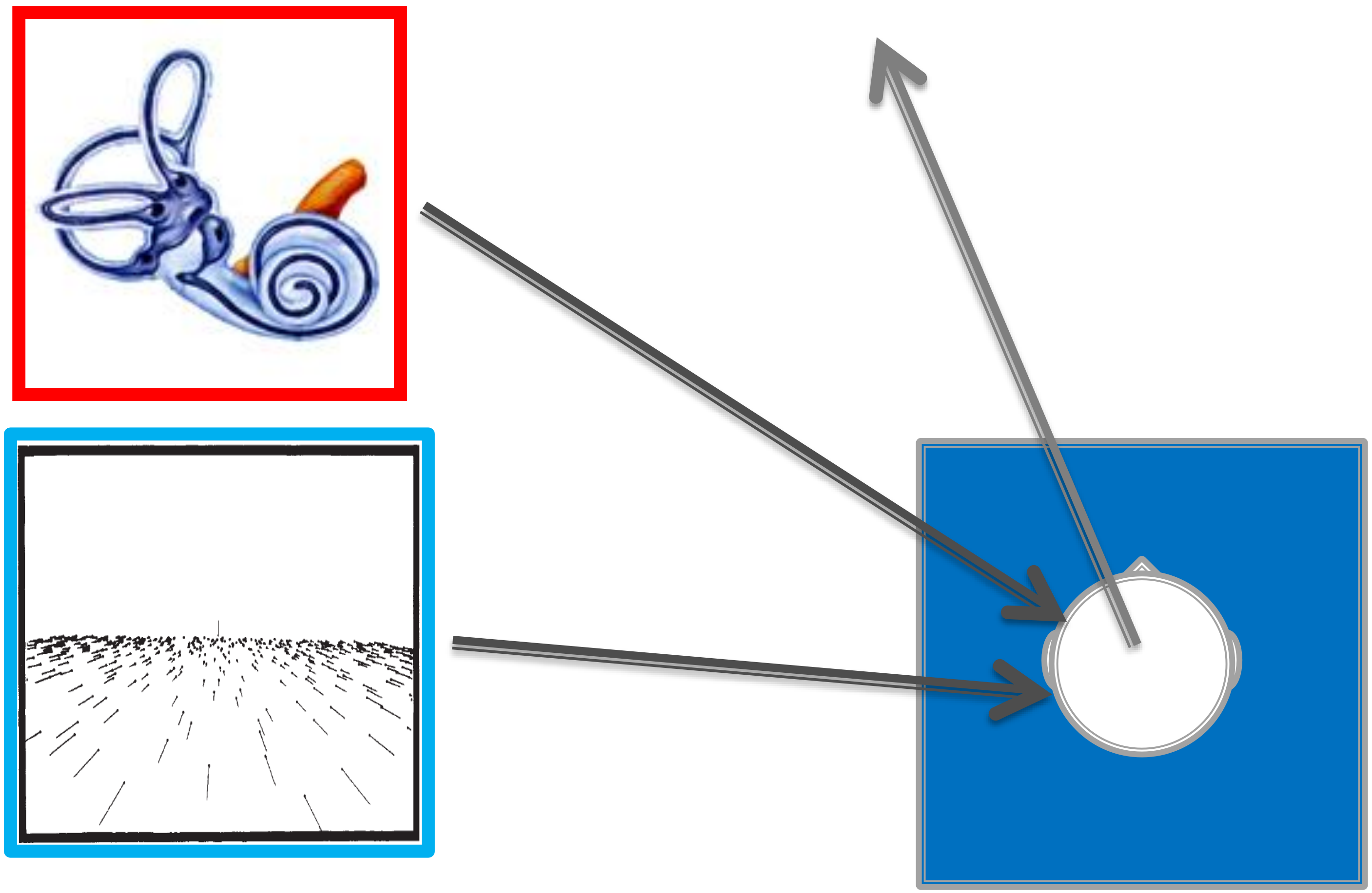
Combination of Senses



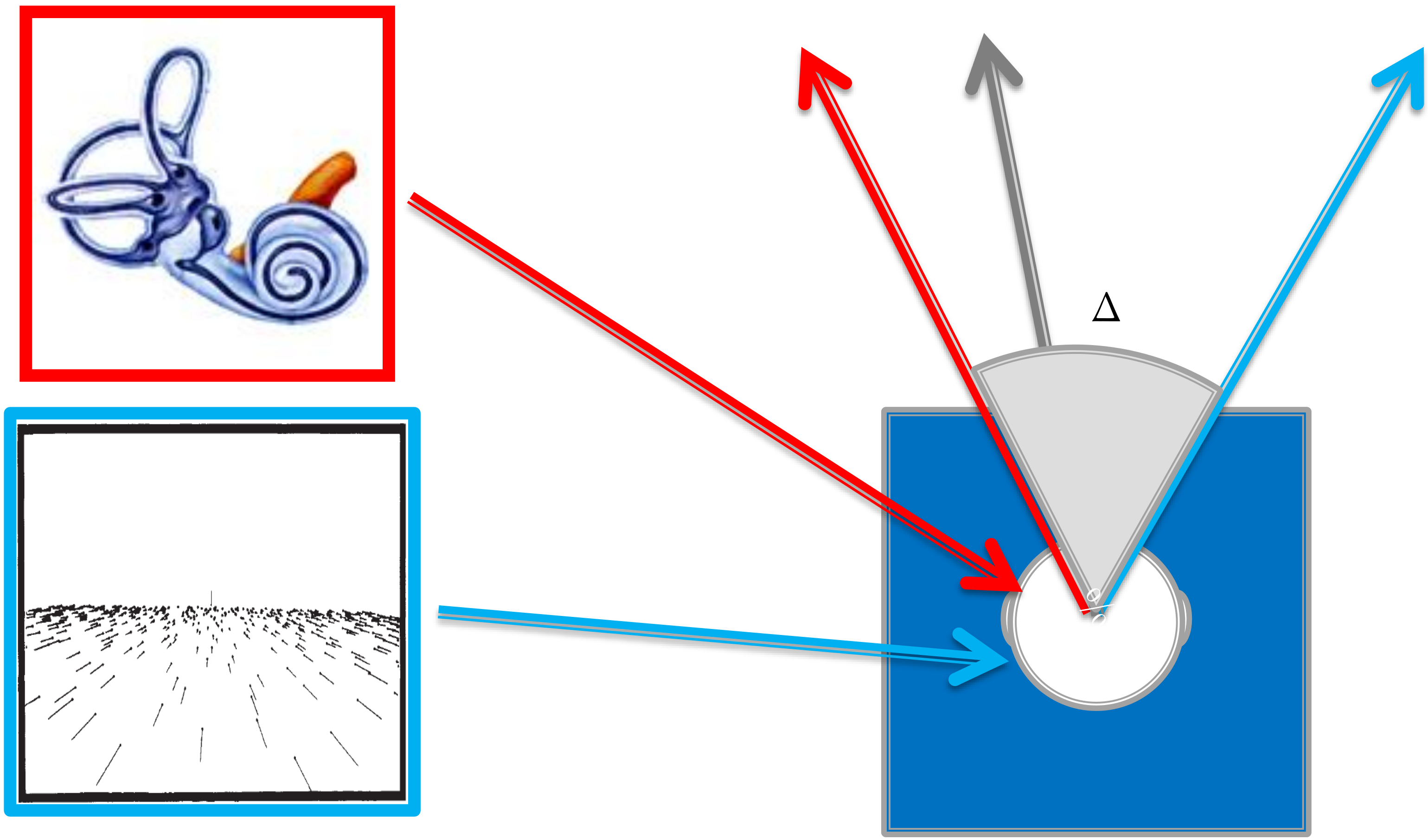


Possible Models

Visual-Vestibular Integration for Heading



Visual-Vestibular Integration for Heading (conflict)

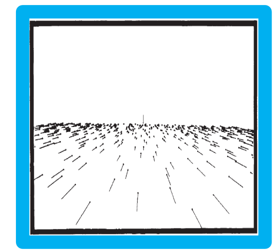


Why introduce a conflict?

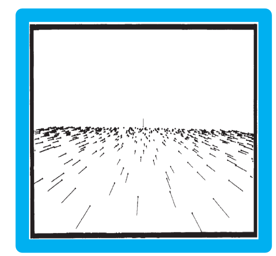
- By introducing a conflict we can see if there is a breakdown of the combination of sense
- We can calculate the weights given to each cue
- To model the observed combined response from the visual and vestibular response

The logic of conflicts

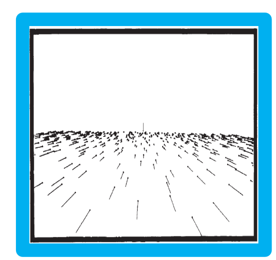
Equally weighted



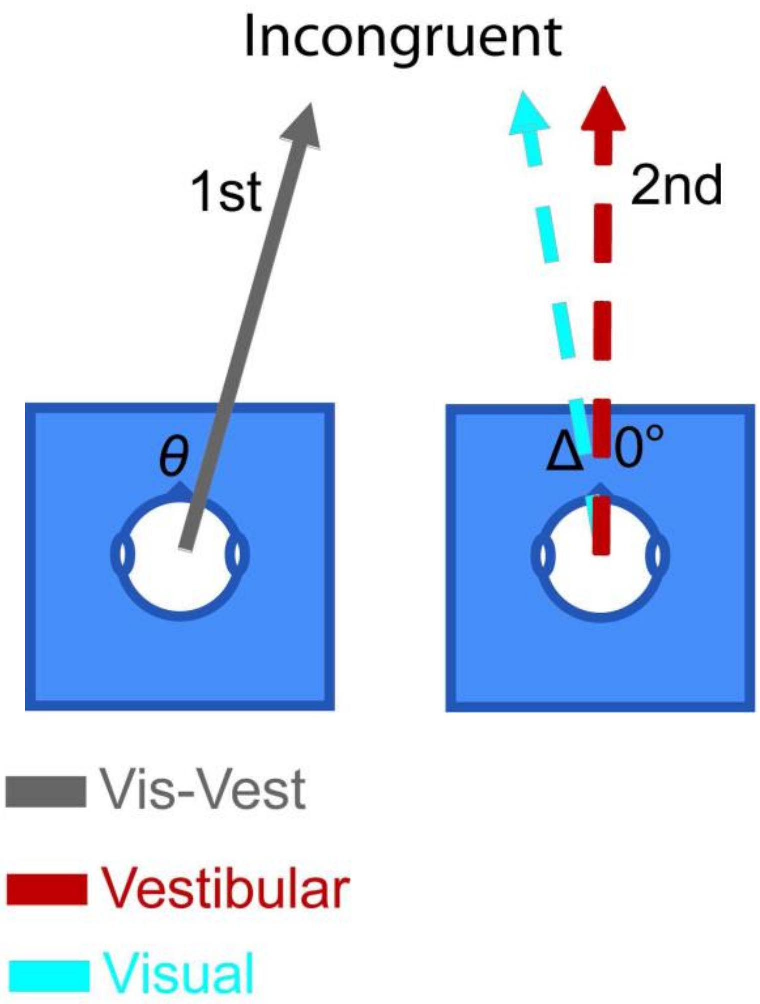
Vestibular weighted more

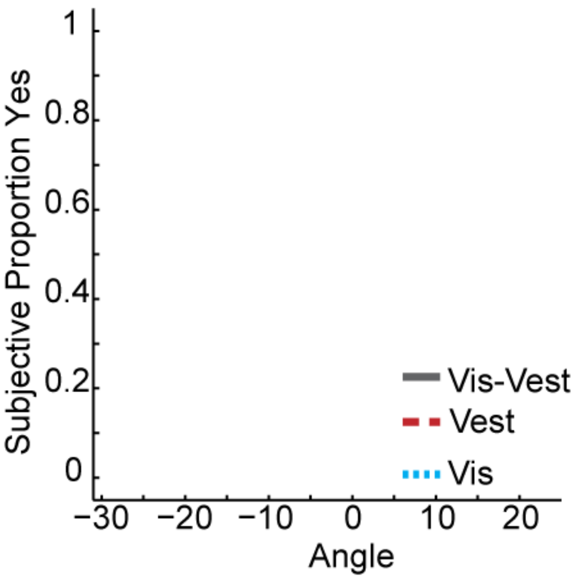


Vision weighted more



Individual participant analysis





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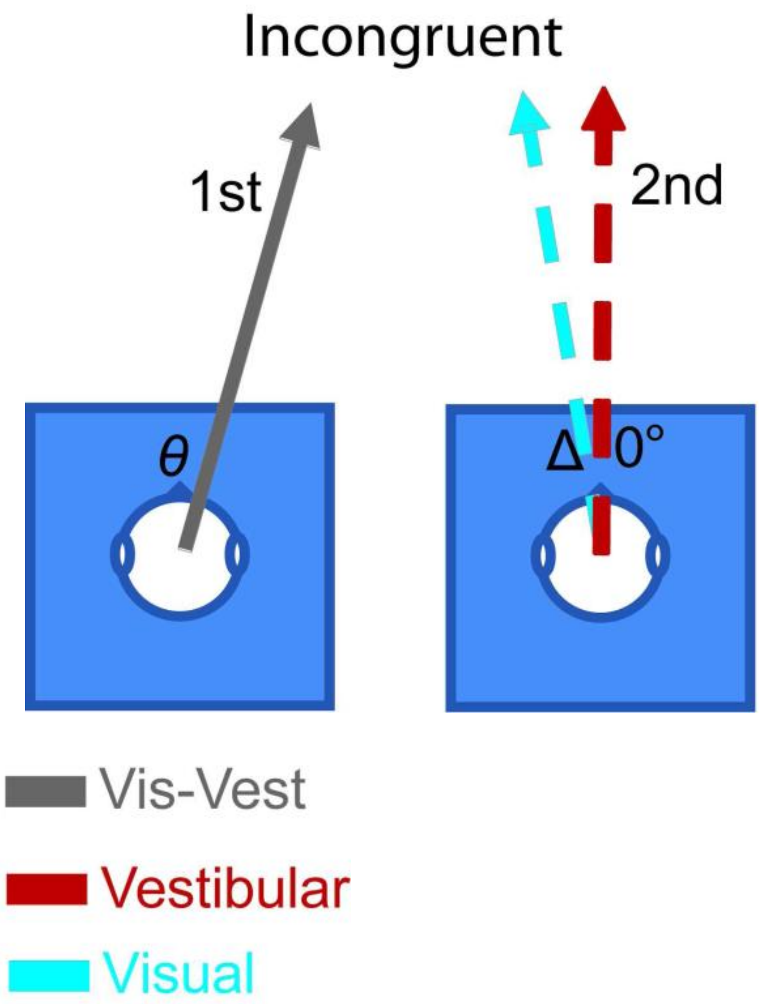

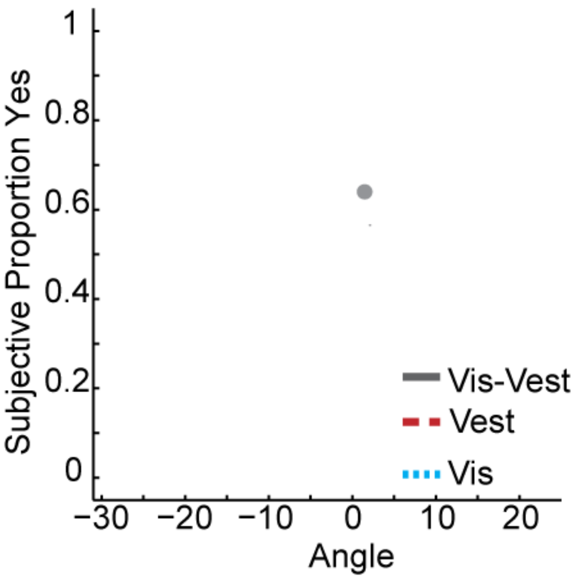

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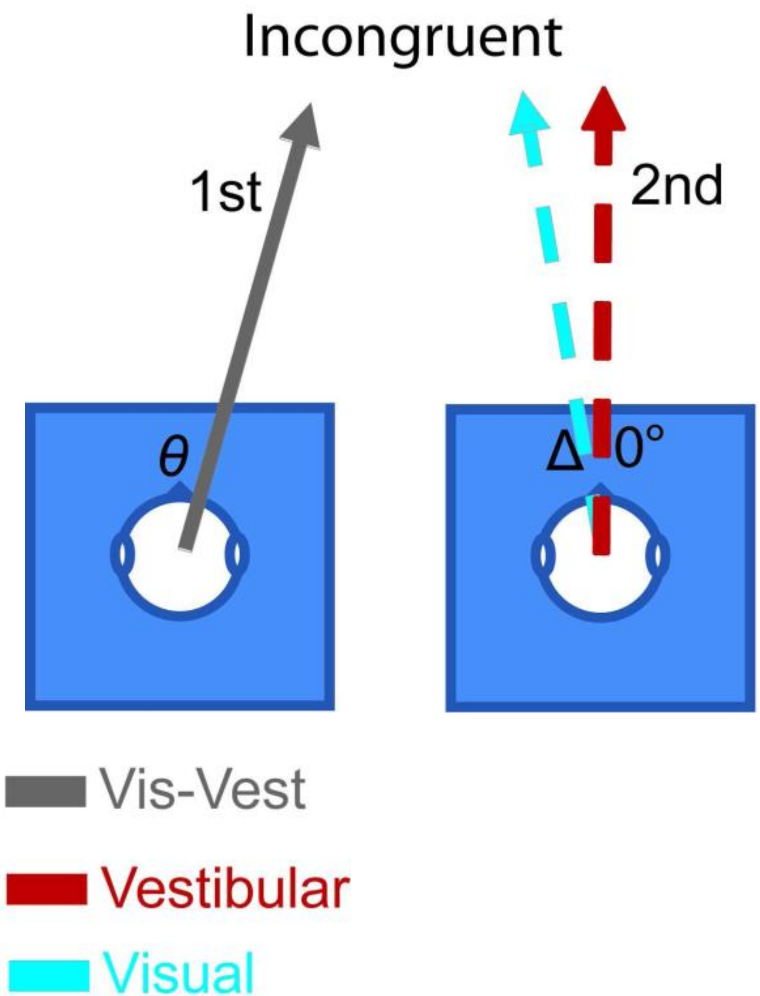

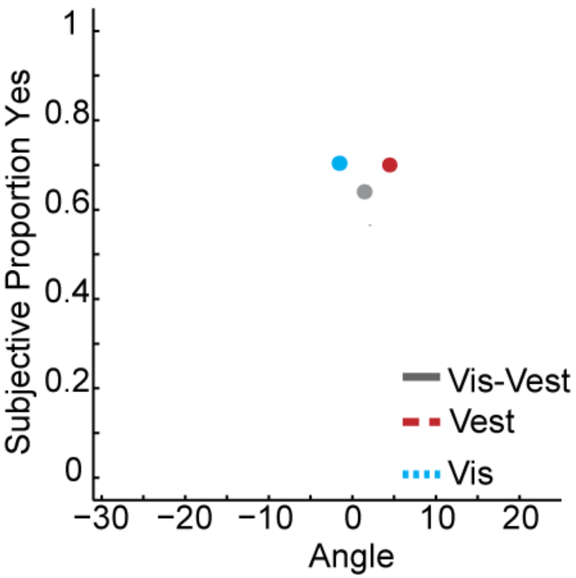

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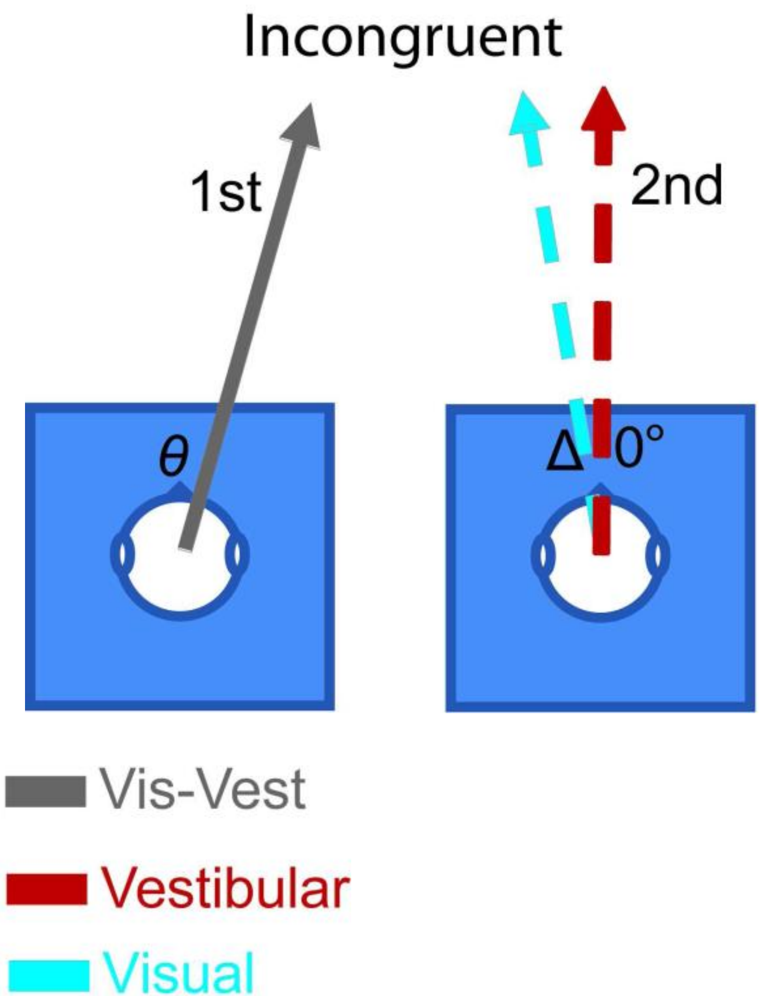

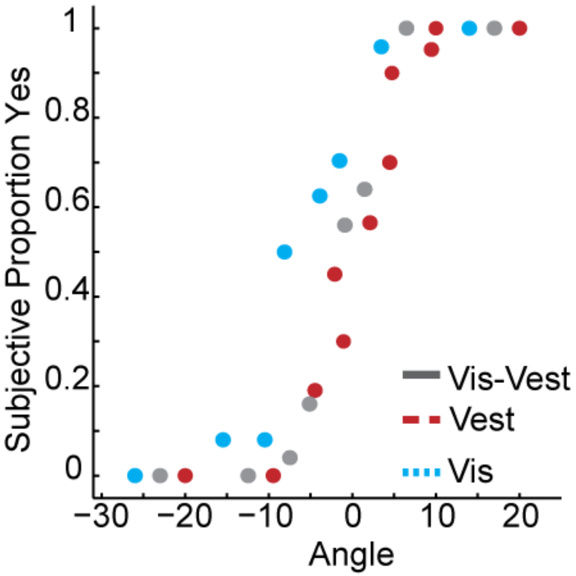

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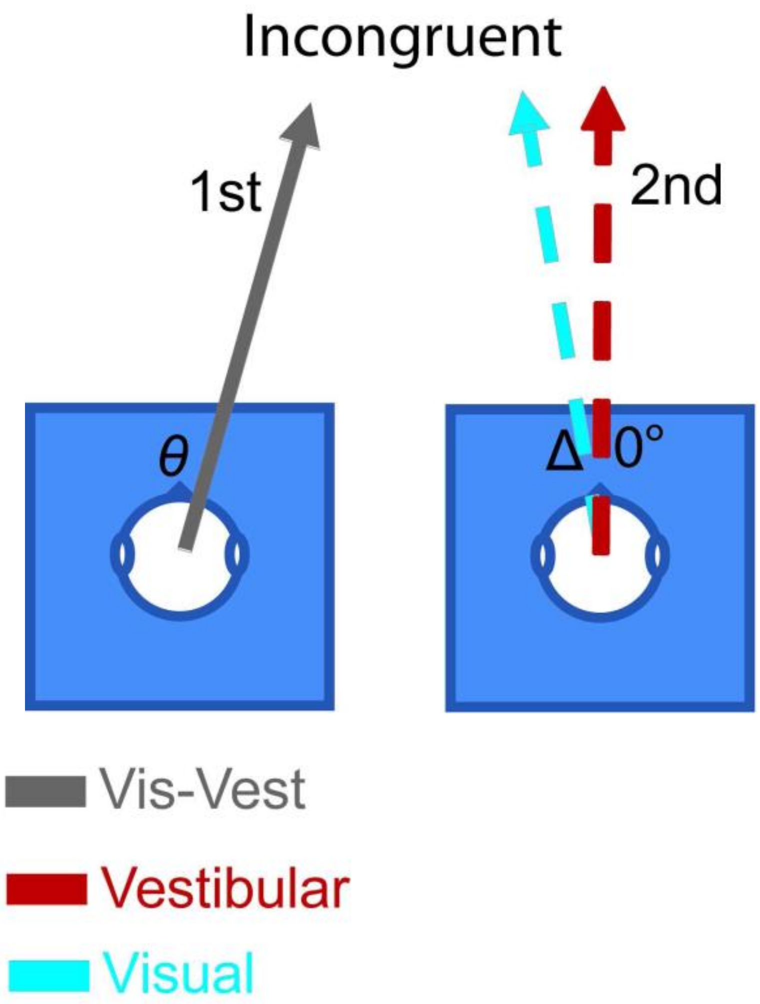
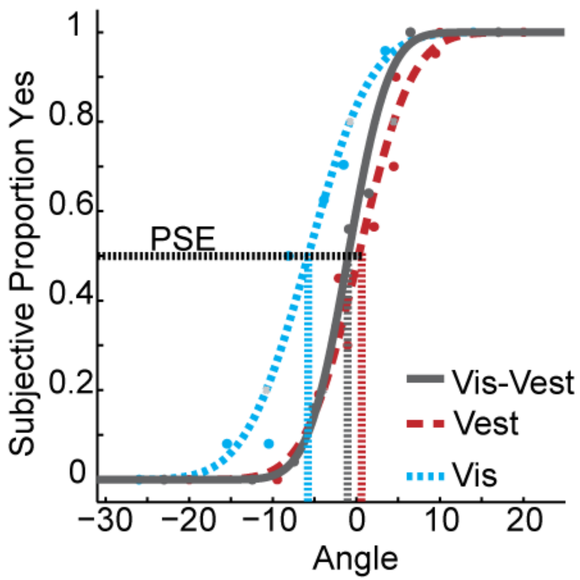




Combination of Senses

WINNER TAKES ALL

- COMBINED the better sense

OPTIMAL

$$JND_{Vis-Vest} = \sqrt{\frac{JND_{Vis}^2 \, JND_{Vest}^2}{JND_{Vis}^2 + JND_{Vest}^2}}$$

Maximum Likelihood Estimation

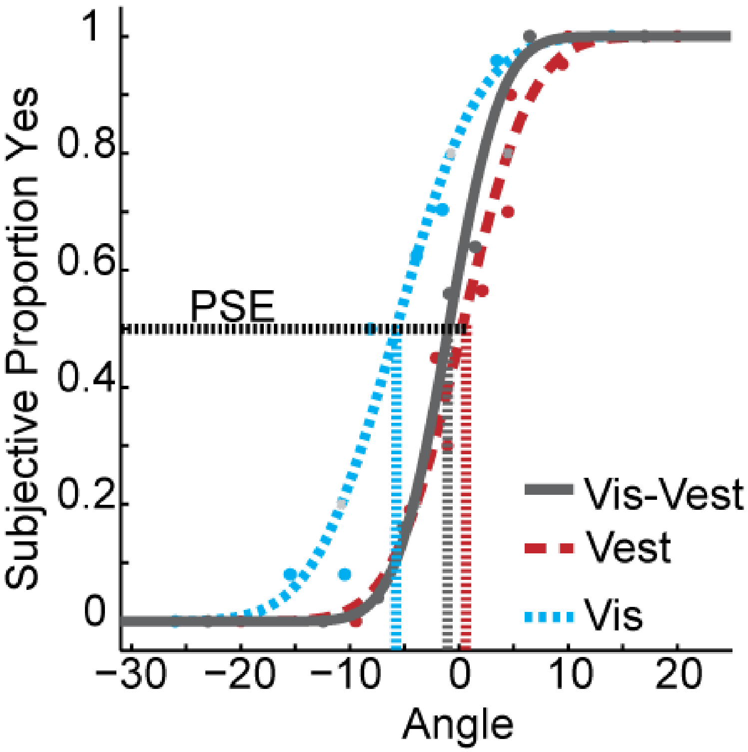
$$\widehat{S}_{Vis-Vest} = w_{Vis}\hat{S}_{Vis} + w_{Vest}\hat{S}_{Vest}$$

Observed

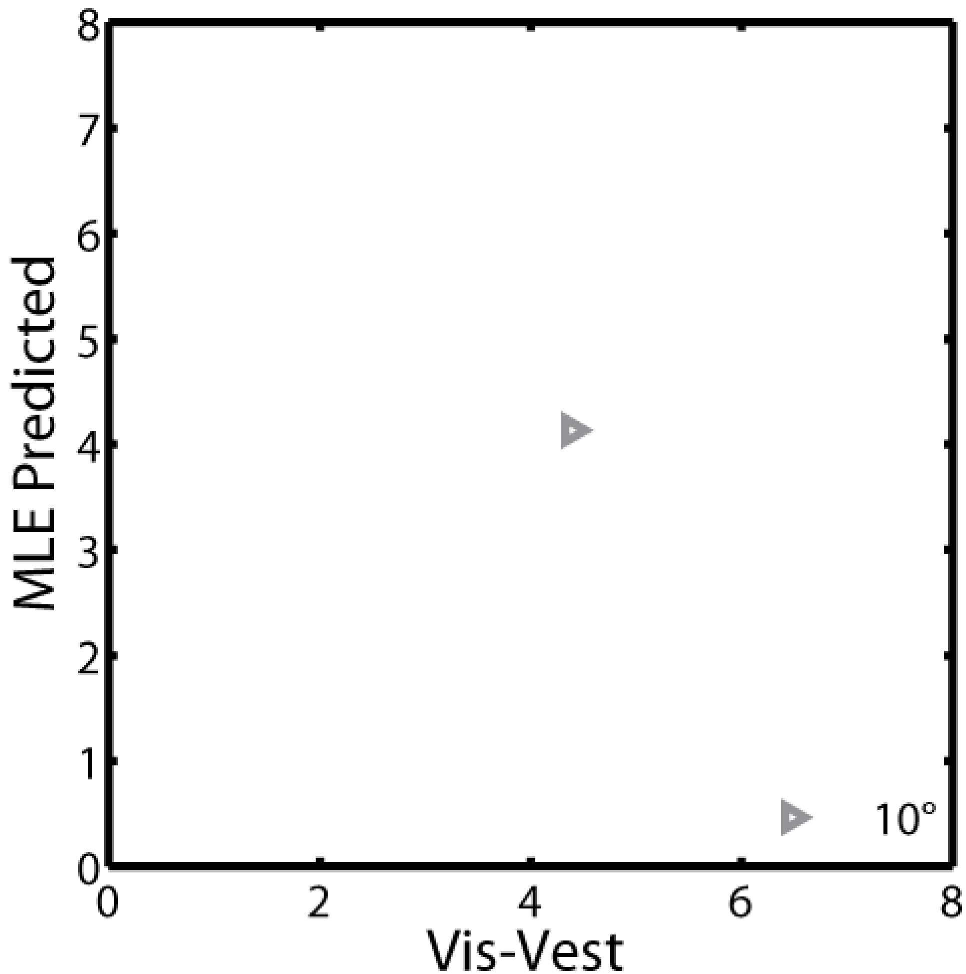
$$w_{Vis} = \frac{PSE_{Vis-Vest} - PSE_{Vest}}{PSE_{Vis} - PSE_{Vest}}$$

Predicted

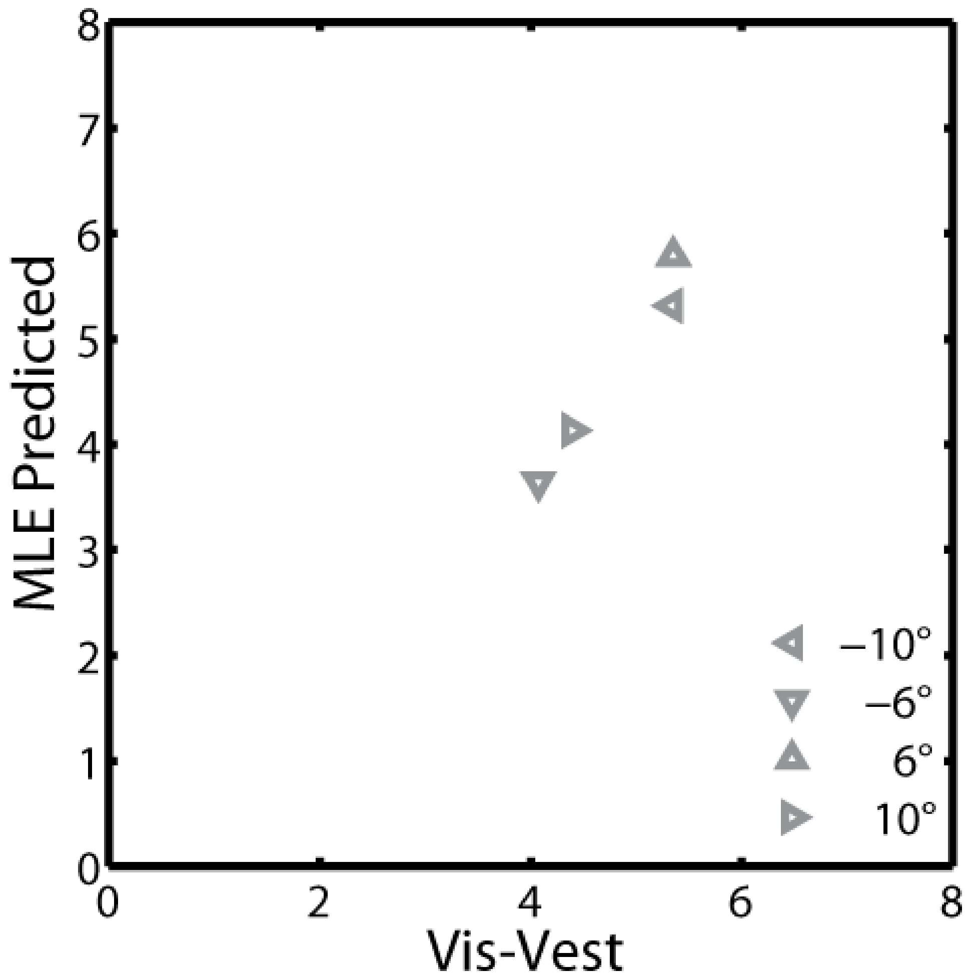
$$\hat{w}_{Vis} = \frac{1/JND_{Vis}^2}{1/JND_{Vis}^2 + 1/JND_{Vest}^2}$$



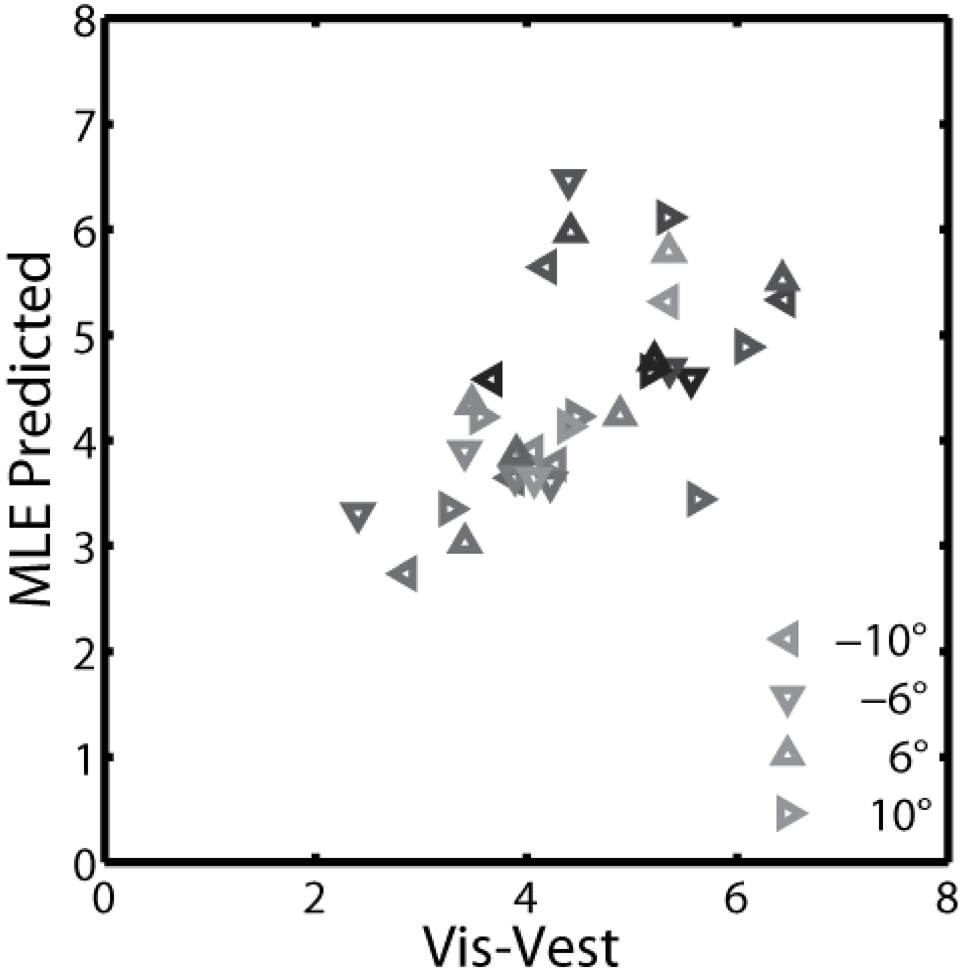
Observed vs Predicted



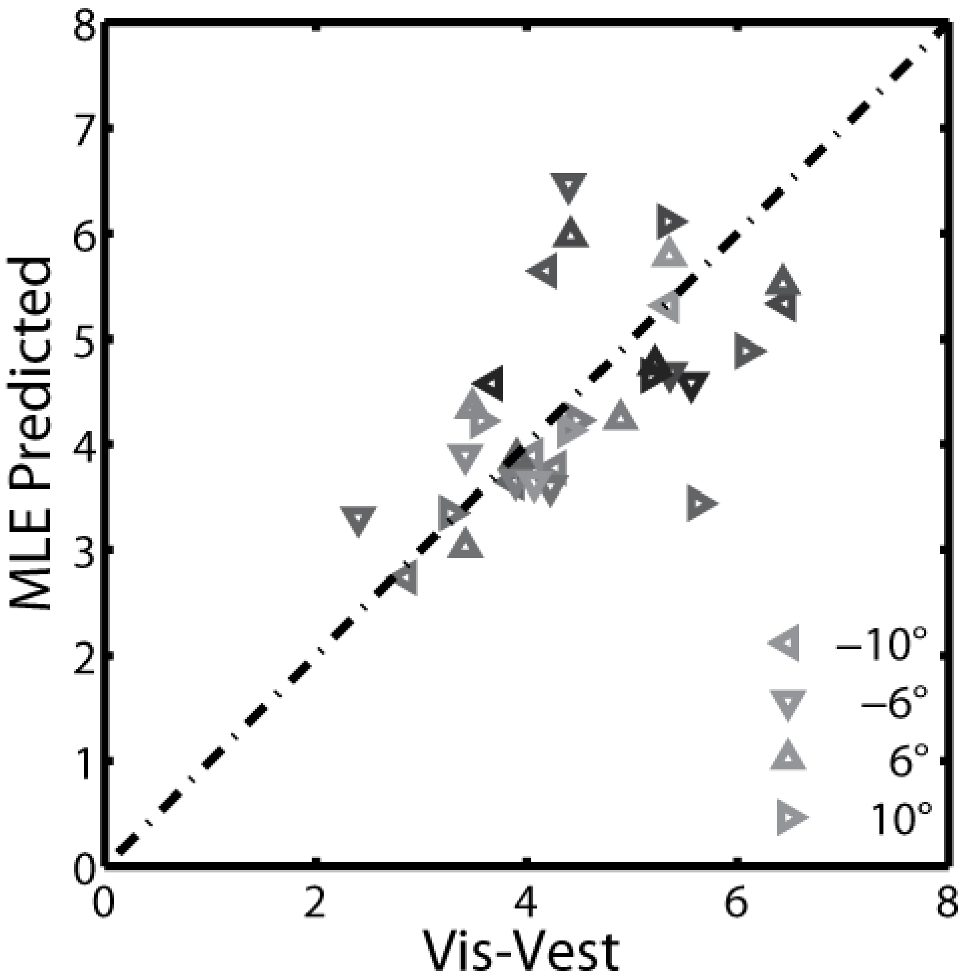
Observed vs Predicted



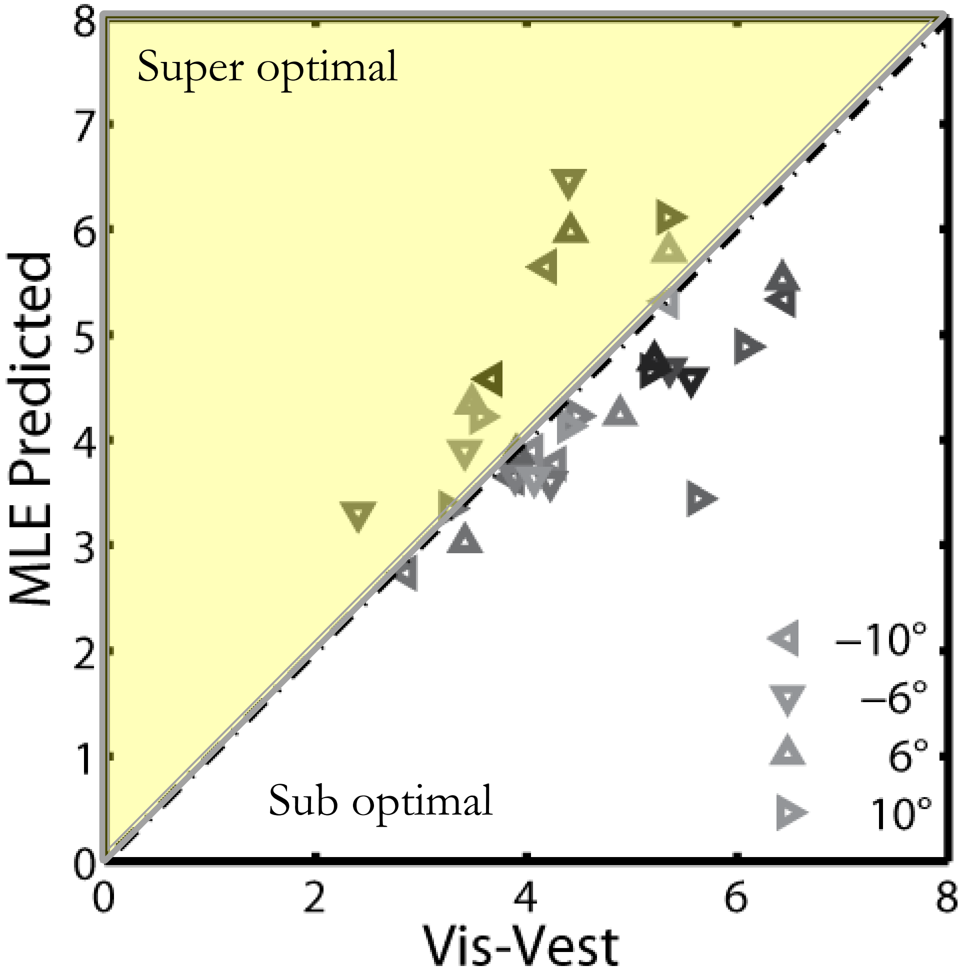
Observed vs Predicted



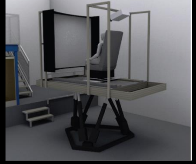
Observed vs Predicted



Observed vs Predicted

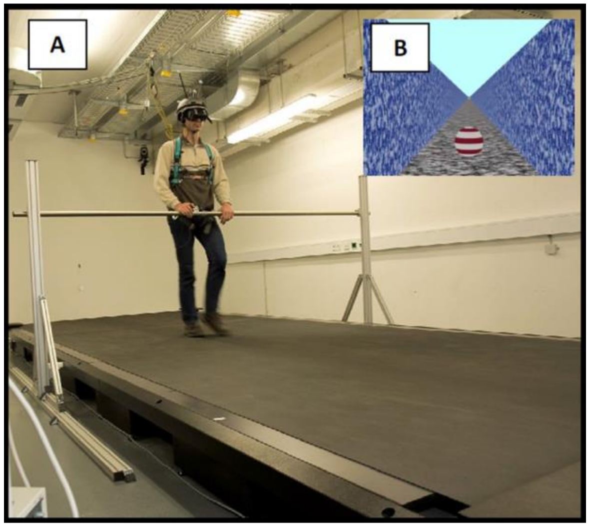


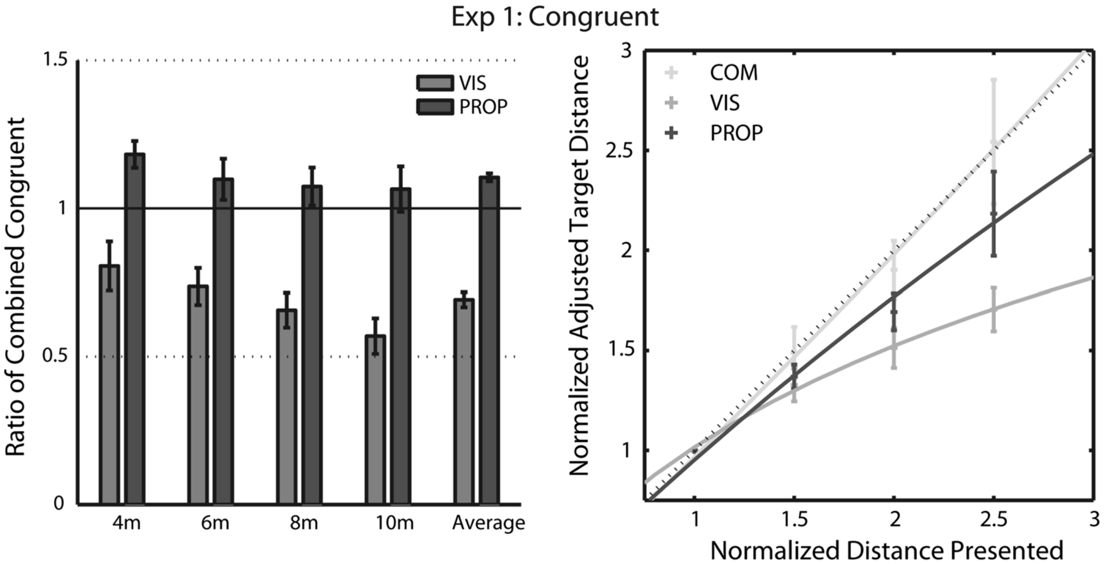
Summary



- The vestibular system is useful
- Sensory information combines in an optimal fashion
- This has also been shown at the neuronal level
- This model extends to most sensory combinations
  - Audio-visual
  - Visual-touch
  - Audio-touch
- Helps explain possible reasons for falls in the elderly

How far did I walk?





$$\frac{dp}{dx} = -\alpha p + k$$

Any questions